A Simple Technique for Calculating the Propagation Dispersion of Multiconductor Transmission Lines in Multilayer Dielectric Media

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Abstract—**A multiconductor transmission line in a multilayer dielectric medium is complicated and therefore is frequently analyzed by the simple quasi-TEM approach. Unlike the full wave eigenvalue approach, the quasi-TEM approach does not give the propagation dispersion characteristics of the line. This paper overcomes this problem by first obtaining the solution of each multiconductor mode from the quasi-TEM approach. Then these solutions are used as both basis and testing functions in a variational formulation of the full wave eigenvalue analysis. The result is high accuracy in the dispersive propagation constants of the multiconductor modes. By solving only for high frequency eigenvalues, not for the high frequency eigenvectors, the method is simpler and faster than the conventional full wave dispersion analyses.**

I. Introduction

RECENTLY, quasi-TEM analysis has been a prominent tool in characterizing multiconductor transmission lines in a multilayer dielectric media [1]–[4]. Quasi-TEM analysis normally includes two steps. Firstly, the distributed circuit parameters, such as the capacitance matrix $[C]$ and the inductance matrix $[L]$, are calculated numerically using an electrostatic approach [1]–[3]. Then the quasi-TEM wave propagation constants and characteristic impedances of all the distinct modes are deternined by solving a coupled set of ordinary differential equations [4], [8].

The propagation constants obtained from quasi-TEM analysis are inaccurate in analyzing high frequency microwave integrated circuits and some high speed digital circuit boards. In these cases, knowledge of the dispersion characteristics of the propagation constants is necessary. By improving the quasi-TEM solutions through a full wave eigenvalue equation, this paper studies the frequency dispersion of all the quasi-TEM distinct modes supported by the multiconductor transmission lines in multilayer dielectric media.

Although some available methods, such as the spectral domain method, the method of lines and the spatial Green's function methods [5]–[7], can be used for obtaining the dispersion characteristics of multiconductor multilayer transmission lines, they are computationally intensive. In addition, these methods may generate some non-physical modes in solving multiconductor transmission line problems, since the resultant matrix size is usually much greater than the number of strip conductors. This paper presents a new technique, which improves the computational efficiency and eliminates the non-physical modes, in determining the propagation constants of the multiconductor modes. In this technique, the charge distribution for each distinct mode is determined first using a quasi-TEM (2-D electrostatic) analysis. This charge distribution is then taken as an approximation of the current distribution for the distinct mode. By using the Galerkin's method *twice* (in Section III), a full wave eigenvalue equation is derived involving the propagation constant and the corresponding current distribution. Therefore the dispersive propagation constant for each distinct mode, at a certain frequency, can be obtained numerically by solving the eigenvalue equation.

For the dispersion characteristics of N_c distinct modes on an N_c-conductor transmission line, our technique has two major advantages over the commonly used eigenvalue techniques reported in [5]–[7], [10], [14]. Firstly, the new eigenvalue equation has only one solution for a pre-determined current distribution of each distinct mode. Therefore the non-physical modes are absent. Secondly, the new eigenvalue equation of full wave analysis involves only $O(N_s^2)$ operations in each iteration step for solving the eigenvalue equation, where N_s is the total number of segments in moment method solution. This is opposed to the commonly used full wave techniques [5]–[7], [10], [14] where $O(N_s^3)$ operations are needed in each iteration step. Therefore, the computational efficiency is improved.

In this paper we only consider infinitely thin strip conductors in carrying out the calculations. The same technique is applicable to conductors of arbitrary cross section.

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II. Low Frequency Charge Distributions by the Quasi-TEM Analysis

As shown in Fig. 1, the transmission line structure studied in this paper consists of two optional ground planes, three optional dielectric layers, and N_c infinitely thin strip conductors, each horizontally located at any position. In this section, we look for the charge distributions on each strip, for all the multiconductor modes. As the first step, each strip is divided into a few smaller segments. The total number of *segments* on all strips, N_s, is usually much greater than the total number of *strips*, N_c. The charge distribution under a certain voltage excitation can be obtained numerically by a two dimensional static moment method in the transverse dimension [1]–[4], [12]:

$$[q_m] = [c_{mn}][v_n], \qquad m, n = 1, 2, \cdots N_s \tag{1}$$

where $[q_m]$ and $[v_n]$ are the charge and the voltage on each segment respectively, and $[c_{mn}]$ is the coefficient matrix of size $N_s \times N_s$ in the moment method solution. The capacitance matrix $[C]$ of $N_c \times N_c$ for multiconductor lines can be obtained by finding the charge distributions on all strips through (1). The inductance matrix of $N_c \times N_c$ can be obtained from the following equation:

$$[L] = \mu_0 \epsilon_0 [C_0]^{-1} \tag{2}$$

where $[C_0]$ is the capacitance matrix found when all dielectric layers are replaced by free space.

It is well known [8] that there are N_c distinct quasi-TEM modes on the transmission line system. With the capacitance and inductance matrices obtained, the N_c distinct modes can be obtained by solving the following telegraph equation:

$$\frac{d^2[V_n]}{dz^2} + \omega^2[LC][V_n] = 0, \qquad n = 1, 2, \cdots N_c \tag{3}$$

where $[V_n]$ is a column vector and represents the voltages of all strip conductors with respect to the ground plane, and $[LC]$ is the product of the inductance matrix $[L]$ and capacitance matrix $[C]$. Equation (3) is a set of N_c *coupled* ordinary differential equations. Through the following linear transformation,

$$[V_n] = [M_{mn}][e_n], \qquad m, n = 1, 2, \cdots N_c \tag{4}$$

where $[M_{mn}]$ is the transformation matrix obtained through the diagonalization of matrix $[LC]$, [15], and $[e_n]$ is the transformed column vector of voltages, (3) can be changed to the following *decoupled* equation set:

$$\frac{d^2[e_n]}{dz^2} + [\beta^0][e_n] = 0 \tag{5}$$

where

$$[\beta^0] = \begin{pmatrix} \beta_1^0 & 0 & \cdots & 0 \\ 0 & \beta_2^0 & \cdots & 0 \\ & & \cdots & \\ 0 & 0 & \cdots & \beta_{N_c}^0 \end{pmatrix} \tag{6}$$

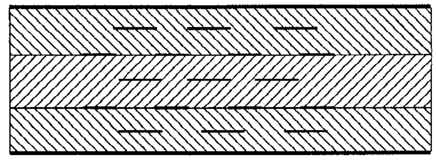

Fig. 1. Multiconductor transmission lines in multilayered media.

and $\beta_n^0 (n = 1, 2, \cdots N_c)$ are known as the propagation constants of N_c distinct quasi-TEM modes on an N_c-conductor transmission line system. The superscript "0" means that these propagation constants are obtained from a quasi-TEM analysis.

In (4), the nth column of $[M_{mn}]$ is a *voltage eigenvector* corresponding to the propagation constant β_n^0. By taking this voltage eigenvector as the enforced voltage excitation on the multiconductor transmission line, and through (1), we can get the charge distribution on each strip for the distinct mode with propagation constant β_n^0. It is noted that although the vector eigenvector has only N_c elements representing the N_c strips, the voltage excitation vector $[v_n]$ used in (1) has N_s elements representing N_s segments over all strips. The charge distribution $[q_m]$ obtained from (1), under an eigenmode voltage excitation, will be used in determining the frequency dispersive propagation constant as discussed in the following section.

III. High Frequency Propagation Constant Through a Full Wave Variational Formulation

To determine the high frequency propagation constant for each distinct mode, we start with the following full wave integral equation [7], [10]:

$$\vec{E}_{\tan}(x) = \sum_{n=1}^{N_c} \int_{S_n} \overline{\overline{G}}_E(x, x') \cdot \vec{J}_s(x')\, dx' = 0 \tag{7}$$

where x denotes the position, $\vec{E}_{\tan}(x)$ is tangential electric field on the strip conductors, $\overline{\overline{G}}_E(x, x')$ is two dimensional dyadic Green's function of the electric field, and $\vec{J}_s(x')$ is surface electric current on the strip conductors. The full wave Green's function can be evaluated accurately using the complex image technique [18].

For the N_c distinct modes, the electric current transversely flowing on the strip conductors is negligible compared to the longitudinal current, i.e., $\vec{J}_s(x') \approx \hat{z}J_{sz}(x')$. This assumption is also used by Knorr *et al.* [9] in solving single and coupled microstrip lines. Since only one component of the electric current is involved in the integral equation (7), we need only match one component of the electric field on the strip conductors, i.e.,

$$E_z(x) = \sum_{n=1}^{N_c} \int_{S_n} G_E^{zz}(x, x') J_{sz}(x')\, dx' = 0. \tag{8}$$

Using Galerkin's method, the integral equation (8) can be converted to the following matrix equation, similar to [7], [10]:

$$[Z_{mn}(\beta)][I_n] = 0, \qquad m, n = 1, 2, \cdots N_s \tag{9}$$

where N_s is the total number of segments on all the strip conductors, $[I_n]$ represents the unknown coefficients of current expansion.

The usual way of finding the unknown propagation constant β from (9) is to search the zeros of the determinant of the matrix $[Z_{mn}(\beta)]$ [5]–[7], [10], [14], using a numerical iteration algorithm. It is known that calculating the determinant of an $N_s \times N_s$ matrix involves $O(N_s^3)$ operations. For a transmission line system composed of one or two strip conductors, calculating the determinant in each iteration step is not a major problem, because the matrix size is small. For multiconductor transmission lines composed of, say, more than five conductors, calculating the determinant of a large matrix takes a large amount of computer time. Moreover, some unphysical modes may be generated because the matrix size is much greater than the number of strip conductors. For instance, a single microstrip line has only one distinct mode. If we use three segments in moment method solution, two non-physical modes will be generated. To reduce the computer time and eliminate the non-physical modes, we propose a new scheme to obtain the frequency dispersive propagation constant for each distinct mode, as described below.

Suppose that the eigenvector $[I_n]$ is known, by pre-multiplying (9) by the transpose vector $[I_n]^T$, we have the following 1×1 matrix equation,

$$[I_n]^T [Z_{mn}(\beta)] [I_n] = 0. \tag{10}$$

The above equation can be considered as the result of re-using the Galerkin's method, where the eigenvector $[I_n]$ is taken as a testing function. The eigenvalue equation (10) is a 1×1 matrix equation, or a transcendental equation. The unknown eigenvalue (i.e., the propagation constant) corresponding to the eigenvector $[I_n]$ can be obtained by solving (10) iteratively.

It is seen in (10) that only a matrix multiplication is involved in each iteration step, which takes $O(N_s^2)$ operations for an $N_s \times N_s$ matrix. This is opposed to the commonly used eigenvalue techniques [5]–[7], [10], [14] where $O(N_s^3)$ operations are needed for calculating the determinant of a matrix in each iteration step. Another favorable feature of (10) is that for a given current distribution $[I_n]$ of an eigenmode, there is only one solution of the eigenvalue. This is again opposed to the commonly used eigenvalue techniques [5]–[7], [10], [14] where N_s eigenvalues exist for an $N_s \times N_s$ matrix. The elimination of the non-physical modes is of special importance in solving multiconductor lines, since it is not an easy task to choose N_c useful eigenvalues out of the total N_s eigenvalues.

Since Galerkin's method is used twice in deriving the matrix equation (9) and the eigenvalue equation (10), the error existing in the eigenvector $[I_n]$ only causes a very small *second order error* in obtaining the propagation constant. This argument is supported by the proof of variational property of Galerkin's method [16]. Numerical results in the next section will further confirm the accuracy of (10).

In solving (10), the rigorous eigenvector $[I_n]$ is usually unknown. In this paper we take the charge distribution $[q_m]$ in (1) as an approximate current distribution, by using the following relation between charge and current:

$$[I_n] = \frac{\omega}{\beta}[q_n], \qquad n = 1, 2, \cdots N_s. \tag{11}$$

The charge distribution $[q_n]$ was obtained from (1) *under the excitation of an eigenmode voltage* as discussed in the last section. In carrying out the calculations of (10), the constant ω/β can be dropped.

The validity of the approximate current distribution obtained from (11) can be argued as follows:

(*a*) On a zero-thickness strip conductor of infinite conductivity, the surface charges and the longitudinal surface current have the same distribution. This is easily seen from the current continuity equation, $\nabla \cdot \vec{J}_s = j\omega\rho_s$, and was also used by Hashimoto [17].

(*b*) The charge distribution $[q_m]$ obtained from quasi-TEM analysis is a good approximation to the true distribution of time-varying charges, and the longitudinal current.

(*c*) Due to the variational property of Galerkin's method, the error introduced in the current distribution causes only a second order error in the eigenvalue from (10). Therefore the frequency dispersive propagation constant can be determined very accurately from (10), as will be confirmed in the following examples.

As a summary, determining the frequency dispersion of a multiconductor transmission line system involves the following steps:

Step 1. Solve an electrostatic problem to obtain the $[C]$ and $[L]$ matrices, using any one of the techniques presented in [1]–[4], [12].

Step 2. Find the eigenvalues and eigenvectors of the matrix $[LC]$. The eigenvalues are the quasi-TEM wave propagation constants of the distinct modes. They can be taken as the initial values for step 4.

Step 3. Taking one eigenvector of step 2 as the voltage excitation, find the charge distribution on each strip for this specific mode.

Step 4. Taking the electrostatic charge distribution of step 3 as the approximate eigenvector of longitudinal current, find the *corrected* propagation constant from (10).

Step 5. Repeat steps 3 and 4 to get the *corrected* propagation constants for all the distinct modes.

IV. Results for Multiconductor Transmission Line Examples

We first validate the technique proposed in this paper through some simple examples, i.e., the single microstrip line and the coupled microstrip line. Then we present

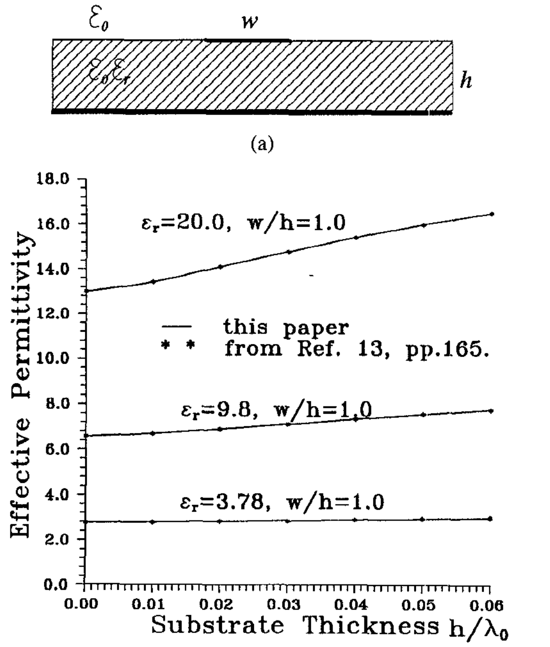

Fig. 2. (a) A microstrip line structure. (b) Frequency dispersion of microstrip lines.

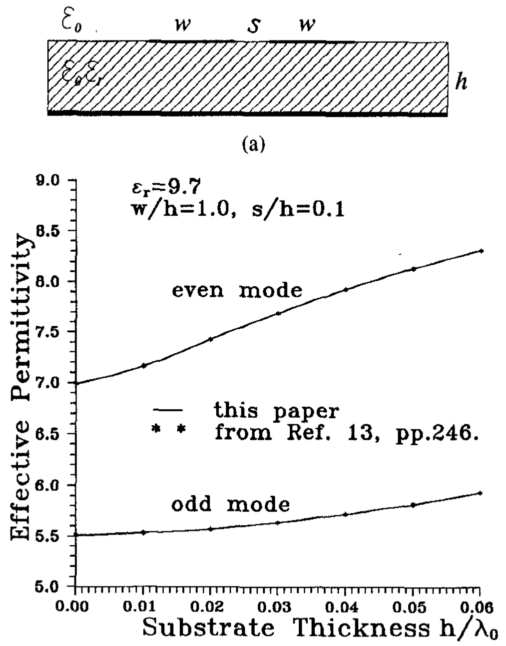

Fig. 3. (a) A coupled microstrip line structure. (b) Frequency dispersion of odd and even modes on coupled microstrip lines.

some results for multiconductor transmission lines in multilayer dielectric media, i.e., a six-strip-coupled microstrip example and a three-strip-coupled inhomogeneous stripline example.

A. A Single Microstrip Line

For a single microstrip line as shown in Fig. 2(a), there is only one distinct mode which is the fundamental mode on the line. We calculated the propagation constants of microstrip lines with the dielectric constants ranging 1.0 to 40.0, the electrical thickness h/λ_0 ranging from 0.0 to 0.15. For 1 mm-thick substrate, this corresponds to the frequency range of dc up to 45 GHz. Our results agree with those given by method of lines [13] with less than 0.2% difference. After converting the propagation constants into effective permittivities, the results are plotted in Fig. 2(b).

It should be emphasized that the current distribution used in (10) is taken to be the same as the charge distribution which is obtained from a quasi-TEM analysis. The results for single microstrip line are excellent. They imply that:

1) For the fundamental mode, neglecting the transverse electric current is valid for the frequency range investigated.
2) The approximate current distribution obtained from a quasi-TEM analysis is sufficient for substitution into (10) to accurately obtain the high frequency propagation constant.

B. Coupled Microstrip: Two Strips

For a two-conductor microstrip line system as shown in Fig. 3(a), there are two distinct modes, i.e., the odd mode with $V_1 = 1$, $V_2 = -1$ and the even mode with $V_1 = 1$, $V_2 = 1$. We calculate the high frequency propagation constants for these two distinct modes using the steps described in last section. Fig. 3(b) shows our results and those obtained using the method of lines [13]. The difference is still within 0.2%.

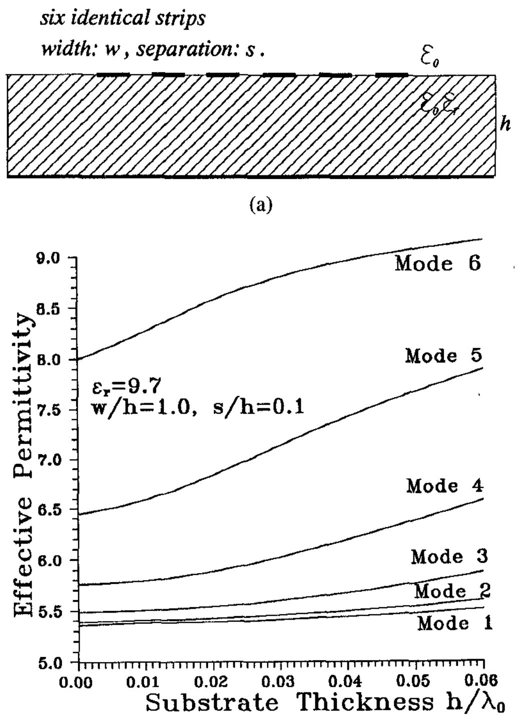

Fig. 4. (a) A coupled microstrip line system with six strip conductors. (b) Frequency dispersion of six distinct modes for the structure shown in Fig. 4(a).

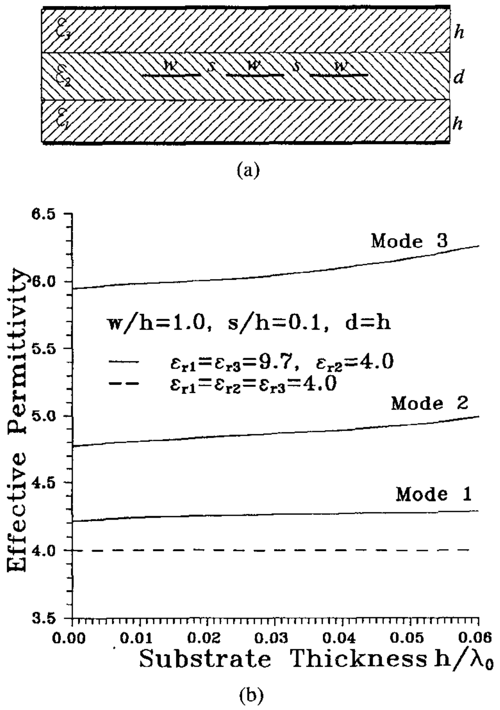

Fig. 5. (a) A three-conductor stripline structure. (b) Propagation constants of three distinct modes for the structure shown in Fig. 5(a).

C. Coupled Microstrip: Six Strips

For coupled microstrip lines with six strip conductors, as shown in Fig. 4(a), the field pattern for each distinct mode is more complicated than that in single or two-coupled microstrip lines. Fig. 4(b) shows the propagation constants of all the six distinct modes.

It is known that in two-coupled line case, the odd mode has a lower effective dielectric constant $\epsilon_{r,\text{eff}}$ than the even mode. There are similarities in the six-coupled line case. The mode with alternatively positive and negative polarities across the strips, designated as mode 1 in Fig. 4(b), has the lowest $\epsilon_{r,\text{eff}}$. The mode with the same polarity across the strips, designated as mode 6, has the highest $\epsilon_{r,\text{eff}}$. Other modes in between are orthogonal to each other and have intermediate $\epsilon_{r,\text{eff}}$.

It is seen in Fig. 4(b) that at high frequencies the true propagation constants for each distinct mode is significantly greater than that from the quasi-TEM analysis.

D. Coupled Striplines: Three Strips with Three Dielectric Layers

The frequency dispersion of multiconductor transmission lines is due to the dielectric inhomogeneity, i.e., the existence of multiple dielectric layers. To show the significance of dielectric layers, we calculate the high frequency propagation constants of a three-conductor stripline system as shown in Fig. 5(a). The propagation constants for three distinct modes are plotted Fig. 5(b), for the homogeneous dielectric case (dotted line) and for the inhomogeneous dielectric case (solid lines) respectively.

It can be seen in Fig. 5(b) that in the homogeneous stripline case, the three distinct modes have the same propagation constant which is equal to $\sqrt{\epsilon_r}k_0$. For the inhomogeneous stripline case, however, three distinct modes are significantly different. All of them have significant frequency dispersion.

V. Conclusion

In this paper, a simple and new technique is presented for calculating the frequency dispersion of multiconductor transmission lines in multilayer dielectric media. In this technique, we first solve a quasi-TEM problem to obtain the quasi-TEM wave propagation constant and the charge distribution on each strip conductor for a distinct mode. Then this charge distribution is taken as an approximation of the current distribution. The latter is used in a new eigenvalue equation to obtain the frequency dispersive propagation constant for that distinct mode. This new technique has two advantages over the commonly used eigenvalue techniques. Firstly, only $O(N_s^2)$ operations are involved in each iteration step for solving the high frequency propagation constant, as opposed to $O(N_s^3)$ operations in each iteration step in the commonly used techniques. Secondly, all the non-physical modes are easily eliminated using the new eigenvalue equation. To validate the new technique, examples of simple and coupled microstrip lines are tested. The results agree with those given by method of lines with less than 0.2% difference. Two more examples, a six-conductor microstrip system and a three conductor inhomogeneous stripline system, are also tested and show significant frequency dispersion.

The emphasis of this paper is a method for obtaining an accurate set of high frequency propagation constants, through a set of low frequency current eigenvectors which are therefore inaccurate at the high frequencies; the method uses a double application of the Galerkin's procedure. After the high frequency propagation constants are accurately obtained from (10), it is possible to substitute them into (9) to obtain the corresponding current eigenvectors, from which the characteristic impedances can be determined. Since this last step is not the main purpose of this paper, and would not be noticeably different from the techniques used in [5]–[7], [12], it is not included in this paper.

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